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Associate Prof. Dr. Arif Emre Dursun

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PREFACE

"Engineering is the art of channeling the great sources of power in nature for the use and convenience of humanity."

Thomas Tredgold

The history of engineering is as old as the sense of curiosity in human beings. Our ancestors worked to use and control the materials and forces of nature for the benefit of humanity, just as we do today. These efforts of engineers have led them to undertake a mission to meet the "needs of the society" within the society. Many of the inventions that illuminate our lives have survived to the present day thanks to the successful efforts of researchers who are devoted to engineering. It is obvious that today's research will also shed light on the future, with the diligent work of valuable scientists. Today, some problems arise in every field from law to economy, from health to technology, and solutions are needed by considering these problems. This book presents the theoretical and applied studies from today's technological perspective, blending them with the latest technological innovations and trends in science and engineering.

Associate Prof. Dr. Arif Emre DURSUN

PROBABILISTIC MODELING OF HYDROLOGICAL DROUGHTS WITH COPULA-BASED MARKOV CHAIN MONTE CARLO SIMULATION

Okan Mert KATIPOĞLU[1], Veysi KARTAL[2],
Chaitanya Baliram PANDE[3,4,5]

INTRODUCTION

Water is an essential source of life and is required for the survival of all life. Effective use and sustainable management of water is of critical importance for humanity. Water scarcity negatively affects all life. Global warming is considered as a significant reason. The studies on climate change clearly show that this is happening all over the world. Millions of years ago, the world climate had a different morphology. Millions of years ago, the greater mass of the average core of the sun caused nuclear reactions in the sun to produce more intense energy by increasing the pressure and temperature in the outer parts of the sun. The drought is an extremely costly natural hazard threatening the sustainability of environmental systems and a

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major impact on a wide range of socio-economic activities, from agriculture to public health. Serinaldi et al. (2009: 34) reported that drought characteristics are often modelled individually with univariate frequency analysis.

Conventional techniques like frequency analysis and univariate modeling were the primary methods used in the past to study droughts. Although these methods are commonly used, they have not been effective in producing precise outcomes due to the random and multifaceted nature of droughts. One effective method for evaluating the potential for drought is by utilizing a probabilistic approach. Hydrological events are complex in nature and are explained by a set of correlated stochastic variables. According to Kim et al., (2006: 31)'s report the use of bivariate distribution has been deemed appropriate for analyzing correlated hydrological random variables. Capula functions are revealed in modeling the relationship of drought characteristics, revealing the dependency structure, and determining their recurrences by taking into account the probabilities of occurrence of the variables. Thus, the drought situation is expressed more comprehensively, allowing managers and planning to make more accurate decisions (Khan et al. 2021: 214; Li et al. 2021: 51).

LITERATURE REVIEW

Due to the complexity phenomenon, the use of a sole character parameter cannot be characterized the comprehensive drought impacts. Two important drought characteristics are often referred to in the literature as randomly associated drought characteristics (Shiau et al., 2012; Shiau and Modarres, 2009: 16). Shiau (2006: 20) and Tosunoglu and Kisi (2016: 543) stated that the method of probability theory is a proper theory to quantitatively study the drought impact. Since drought characteristics are variables that are randomly related to each

other, however, no significant relationships can be found between these variables in univariate analyses of drought characteristics. A better approach is the multivariate analysis feature to assessment of drought characteristics. However, Mishra and Singh (2010: 391) stated that multivariate models are complex to obtain a joint function of the distribution of variables of drought.

Although there are many theories in the literature to identify the joint distribution of drought variables (Salas et al. 2005: 131, Kim et al. 2003: 8, Bonaccorso et al. 2003: 17; Shiau and Shen 2001: 127). Such theories are not applicable due to some difficulties in obtaining their mathematical derivatives and parameters by matching them to the observed data (Chen et al., 2012: 17). Thus, the multivariate and bivariate distributions using copula functions can handle the aforementioned problems. The copula function represents a useful method for analyzing the dependence and the correlation between several variables by relating multivariate distributions to each other based on the univariate distributions (Sklar, 1959: 8). Recently, copulas are a commonly used method for tackling problems of multivariate analysis of hydrological variables (Bárdossy, 2006: 42; Kao and Govindaraju, 2010: 380; Wang, 2016: 82; Ismail et al., 2018: 30; Vazifehkhah et al., 2019: 24; Wang et al., 2019: 11; Chacon-Hurtado et al., 2017: 21). Salvadori et al. (2007: 56) gives a comprehensive and thorough explanation of the theoretical background for the use of copulas.

Using two-dimensional copulas, Shiau et al. (2007: 21) established the joint probabilities and return period for drought duration and severity. Yoo et al. (2016: 11) conducted an analysis of the model success of different copula functions, and based on this analysis they simulated the confidence intervals for the bivariate drought frequency curve. Ahmadi et al. (2018: 77) implemented copula to predict the bivariate frequency analysis

of low Dez River Basin flow in Iran. Song and Singh (2010: 24) generated joint probability distributions of severity, duration, and inter-arrival time of drought employing a trivariate Placket copula. She and Xia (2018: 32) investigated copula-based drought event detection and risk assessments in the Loess Plateau of China, as characterized by SPEI from 1950 to 2014. They found that the northwest of the LP was much more severe and prolonged than the northwest of the LP. The copula based on another study was used by Shiau et al. (2007: 21) to analyze hydrological droughts in the Yellow River in China. The longer the observed drought lasted, the more intense it became, as indicated by a statistically significant association. The marginal distribution of drought characteristics was obtained as bivariate exponential and gamma distributions. The years 1919-2002 and 1930-1933 were reported to be the most severe droughts in the Yellow River. Considered as a whole, China has faced many severe droughts affecting social life, economy and agriculture. Liu et al. (2016: 80) investigated the return period of meteorological drought in China applying copula. The SPEI was employed in this study to determine the drought duration, drought intensity and drought peak between 1961 and 2013. To define the marginal distribution of severity, peak and duration, Weibull, Exponential and Pareto were applied respectively. While Gaussian copula and Clayton copula were used to generate the joint distribution of duration-severity-peak and severity-peak, Gumbel-Hougaard copula was fitted to generate the multivariate distribution of duration-peak and duration-severity, respectively. Reddy and Singh (2014: 28) employed meta-heuristics to predict the copula parameters and applied the methodology to the arid region in Texas, USA. Initially, they performed SPI to characterize drought characteristics and applied copula to quantify multivariate drought risks. The particle swarm and genetic algorithm were employed to

accurately predict the copula parameters. Finally, it is shown that the application of meta-heuristics to predict the copula parameters is much better than other methods. In the literature, there are few studies on Turkey. To illustrate, Tosunoğlu and Can (2016: 82) used copula to simulate the joint probability distributions of drought for the period 1966-2006. They applied SPI to identify drought events and performed principal component analysis to test homogeneity. A variety of bivariate distribution functions were employed to determine a more appropriate marginal distribution of drought characteristics. It was found that Weibull, Gamma and Pareto distributions gave good agreement among the marginal distributions. The optimal copula for each region was obtained by Clayton, Frank, Ali-Mikhail-Haq and Gumbel-Hougaard. Hesami Afshar et al. (2016: 8) evaluated to define the conditional return period at both spatial and temporal scales over Ankara between 1960 and 2013. It was concluded that the normal copula indicates better results than other copulas and there is a stronger relationship between drought characteristics. Vazifehkhah et al. (2019: 24) conducted a bivariate risk analysis of drought by employing copula in the Konya closed basin using 10 meteorological stations. They firstly determined the most appropriate marginal distribution for drought event and obtained log-logistic (severity) and lognormal (duration) distributions, respectively. The bivariate distribution function was simulated with four different copula families and considering the upper tail dependence. Esit and Yuce (2023: 151) defined drought events by standard precipitation index (SPI-3) in Ceyhan Basin in Turkey using bivariate and univariate frequency analysis to simulate the joint return period with copula theory.

In this study it is aimed to obtain probabilistic modeling of hydrological droughts using copula-based approach with Bayesian Inference and Markov Chain Monte Carlo simulation.

MATERIAL AND METHODS

Study Area and Data

The Yeşilırmak Basin is primarily characterized by a semi-arid climate. The northern basin exhibits continental climate attributes, featuring cold and rainy winters, and hot and dry summers. 2. The adverse effects of drought conditions in Yeşilırmak Basin on the environment, water resources, and economy have been highlighted by (Hezarani and Keskin 2021; Zeybekoglu et al. 2023: 123; Katipoğlu, 2023: 15). Montly SDI values of 1401 station of Yesilirmak Basin were calculated. SDI values were determined by drought characteristics and drought recurrence analysis was applied with copula function. In Figure 1, the geographical coordinates of the study area are presented.

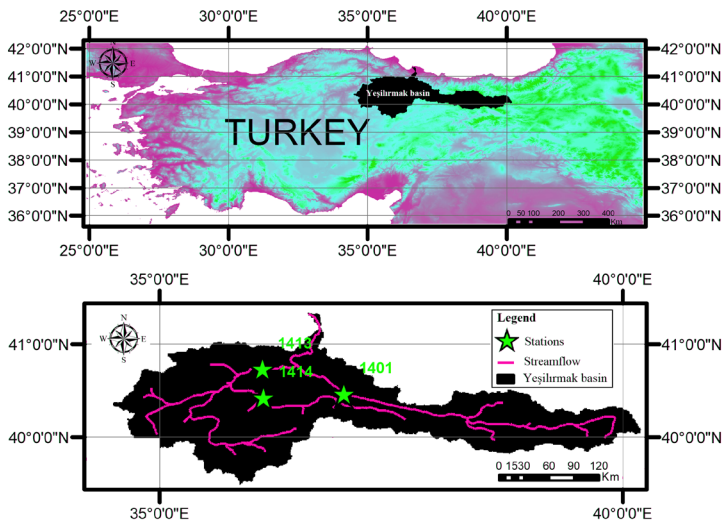


Figure 1: Geometric coordinates of the working area

Streamflow Drought Index Method (SDI)

The SDI method, which was suggested by Nalbantis and Tsakiris in 2009, is an effective way to predict the severity and

frequency of hydrological droughts. The method in question is designed to consider how droughts affect the river basin and incorporate this information into its approach. The availability of a time series of monthly flow volumes Qi, j is a prerequisite for calculating SDI values, where i shows the hydrological year and j indicates the month of the hydrological year. The researchers used total streamflow volumes as a means to investigate the distribution and variation of drought severity across multiple timeframes. From this analysis, they were able to determine the frequency of drought occurrence as well as the cycles and severity of drought in the study region. The total monthly streamflow volume is predicted from successive time series in this way. $V_{i,k}$ is calculated as below:

$$V_{i,k} = \sum_{i}^{3k} Q_{ij} i = 1,2,3,\ldots,12,; k = 1,2,3,4 \quad (1)$$

in which $V_{i,k}$ is the cumulative streamflow volume. Based on $V_{i,k}$, SDI is stated for each related period as below:

$$\mathrm{SDI_{i,k}} = \frac{V_{i,k} - \overline{V_k}}{S_k} = 1,2,3\ldots,12; k = 1,2,3,4 \quad (2)$$

$\overline{V_k}$ and S_k, are the mean and the standard deviation of $V_{i,k}$, respectively. The drought severity categorization of Al-Faraj et al. (2014: 6) was used in the study (Table 1).

Table 1: Drought classification for SDI

Condition	Range
Extremely wet	SDI≥2.00
Very wet	1.50≤SDI<2.00
Moderately wet	1.00≤SDI<1.50
Near normal	-1.00≤SDI<1.00
Moderately dry	-1.50≤SDI<-1.00
Severely dry	-2.00≤SDI<-1.50
Extremely dry	SDI≤-2.00

Univariate Distribution

The definition of drought severity has been commonly attributed to the application of the gamma distribution, as noted by (Chen et al. 2013: 18; Shiau 2006: 20). In accordance with Copula's theorem, the distribution functions in two variables are functions of univariate marginal distributions. For this reason, univariate cumulative distribution functions are first fitted to drought characteristics. The drought duration and severity fit exponential and gamma distributions for hydro-geometric data were widely known (Shiau 2006: 20; Wong et al. 2008: 49).

$$f_S(s) = \frac{s^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-\frac{s}{\beta}}, s > 0 \quad (3)$$

$$f_D(d) = \frac{1}{\lambda} e^{-\frac{d}{\lambda}}, d > 0 \quad (4)$$

in which β shows scale parameter of distribution, λ indicates the exponential distribution parameter acquired from the observed data, α denotes shape parameter of distribution.

Copula Function

Gaussian, Clayton, Gumble, Frank, AMH, Plackett and Joe copulas written in the below are utilized that are available in the literature.

Clayton copula (Clayton, 1978: 65):

$$\max\left(a^{-\theta} + b^{-\theta} - 1, 0\right)^{-\frac{1}{\theta}}, \theta_2 \in \frac{[-1,\infty)}{0} \quad (5)$$

Gaussian copula (Li et al. 2013: 49):

$$\int_{-\infty}^{\phi^{-1}(a)} \int_{-\infty}^{\phi^{-1}(b)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left(\frac{2\theta xy - x^2 - y^2}{2(1-\theta^2)}\right) dxdy, \ \theta \in [-1,1] \quad (6)$$

Frank copula (Li et al. 2013: 49):

$$-\frac{1}{\theta} In\left(1 + \frac{(exp(-\theta a)-1)(exp(-\theta b)-1)}{exp(-\theta)-1}\right), \theta \in \mathbb{R}\backslash 0 \quad (7)$$

Gumbel copula (Li et al.2013: 49)

$$exp\left(-\left(\left((-In(a))^{\theta}\right)+\left(-In(b)\right)^{\theta}\right)^{\frac{1}{\theta}}\right), \theta \in [1,\infty) \quad (8)$$

Joe copula (Esit and Yuce 2023: 151):

$$1-\left[(1-a)^{\theta}+(1-b)^{b}-(1-a)^{\theta}(1-b)^{\theta}\right]^{\frac{1}{\theta}} \quad (9)$$

AMH copula (Kumar, 2010)

$$\frac{ab}{1-\theta(1-a)(1-b)}, \theta \in [-1,1] \quad (10)$$

Plackett copula (Yee et al. 2014: 1605)

$$\frac{\left[1+(\theta-1)(a+b)-TermA^{\frac{1}{2}}\right]}{[2(\theta-1)]}, TermA=\{1+(\theta-1)(a+b)\}^2-4ab\theta(\theta-1) \quad (11)$$

Markov Chain Monte Carlo (MCMC)

The copula function can be determined by I2(F, G) to I(H), where I is an element of the interval [0,1]. The function maps [F(x), G(y), H(x,y)] into I3. The variables X and Y are continuous random variables, and their distribution functions are F(x) = P(X≤x) and G(y) = P(Y≤y), respectively. H(x,y) = P(X≤x, Y≤y) is a function that notes the joint distribution of X and Y.

An unknown process in all types of copula-based models k is related to $\tilde{Y}$ to q∗ in inference analysis of modelling (Sadegh and AghaKou, 2017: 53) and it can be written as below.

$$\tilde{Y} = \kappa(\theta) + \xi \quad (12)$$

in which ξ refers to measurement error vector. e={e1, e2,⋯, en} in which n is the number of observation The vector $e = \tilde{Y} - Y$ is the error residual (Sadegh and AghaKou, 2017: 53). The technique of Bayesian analysis will be employed for the purpose of model inference and uncertainty quantification. According to Sadegh and AghaKou (2017: 53), this is due to

the use of a probability distribution to quantify uncertainty. The following equations are used to estimate the model parameters according to Bayes' law. (Sadegh and AghaKou, 2017: 53).

$$p(\theta|\tilde{Y}) = \frac{p(\theta)p(\tilde{Y}|\theta)}{p(\tilde{Y})} \quad (13)$$

in which $p(\theta)$ is prior distribution of parameter and $p(\theta|\tilde{Y})$ is posterior distribution of parameter. $p(\tilde{Y}|\theta) = L(\theta|\tilde{Y})$ expresses the likelihood written below:

$$L(\theta|\tilde{Y}) = \frac{n}{2} In \frac{\sum_{i=1}^{n}[\tilde{y}_i - y_i(\theta)]^2}{n} \quad (14)$$

The analysis of equation (14) utilizes a MCMC modeling technique. For further details, the readers are directed to Sadegh and AghaKou (2017: 53).

Return Period

According to Shiau and Shen (2001: 127), one of the general approaches to controlling water resources systems and hydrological and hydraulic facilities is through the estimation of the drought return periods. The drought return periods, in particular, give important information about drought conditions. In order to determine the univariate return period of drought duration greater than or equal to a given value, the following calculation can be performed.

$$T_D = \frac{E(L)}{1 - F_D(d)} \quad (15)$$

in which TD shows the return period identified by drought duration, L indicates expected drought interval time. If a certain value is given, it is possible to obtain the drought severity return period that is greater than or equal to it using the exact same equation. This equation can be expressed in the following formulation:

$$T_S = \frac{E(L)}{1 - F_S(s)} \quad (16)$$

There are two situations in which the duration and severity of a common drought can be predicted: drought return period for S≥s and D ≥d and joint return period for S≥s or D≥d expressed with T′DS and TDS, respectively in the following:

$$T_{DS} = \frac{E(L)}{P(D)\geq d, S\geq s} = \frac{E(L)}{1-F_D(d)-F_S(s)+F_{DS}(d,s)} = \frac{E(L)}{1-F_D(d)-F_S(s)+C(F_D(d),F_S(s))} \quad (17)$$

$$T'_{DS} = \frac{E(L)}{P(D)\geq d \text{ or } S\geq s} = \frac{E(L)}{1-F_{DS}(d,s)} = \frac{E(L)}{1-C(F_D(d),F_S(s))} \quad (18)$$

in which FD(d) and FS(s) are CDFs of univariate of drought duration and severity. Any type of copula is symbolized as C. d' shows the conditional probability of drought severity with drought duration. s' shows the conditional probability. The probabilities of condition are calculated as below:

$$P(S \leq s|D \geq d) = \frac{P(D\geq d', S\leq s)}{P(D\geq d')} = \frac{F(s)-F(d',s)}{1-F(d')} = \frac{F_S(s)-C(F_D(d'),F_S(s))}{1-F_D(d')} \quad (19)$$

In Similar way:

$$P(D \leq x|S \geq s') = \frac{P(D\leq d, S\geq s')}{P(S\geq s')} = \frac{F(d)-F(d,s')}{1-F(s')} = \frac{F_D(d)-C(F_D(d),F_S(s'))}{1-F_S(s')} \quad (20)$$

The conditional states of drought duration and severity return period can be derived for two cases. Firstly, the return period of drought severity can be determined stated a drought duration above a specific threshold. Secondly, the return period of drought duration can be established stated a drought severity above a specific threshold. Both conditional return period defined by Shiau (2006: 20) based on copula theory are indicated below:

$$T_{D|S\geq s} = \frac{T_S}{P(D\geq d, S\geq s)} = \frac{E(L)}{[1-F_S(s)][1-F_D(d)-F_S(s)+C(F_D(d),F_S(s))]} \quad (21)$$

$$T_{S|S\geq d} = \frac{T_D}{P(D\geq d, S\geq s)} = \frac{E(L)}{[1-F_D(d)][1-F_D(d)-F_S(s)+C(F_D(d),F_S(s))]} \quad (22)$$

Performance Metrics

Maximum Likelihood Method was defined as a solution to the parameter estimation problem to solve estimation problem. The advantages of this method include extracting all relevant information from the data and providing accurate and efficient estimates for fitting probability distributions to the data (Thyer et al., 2009: 45; Schwarz, 2011: 57).

Consider the population sample $y = (y1, y2, \ldots, yi, \ldots, yn)$. Note that $f(yi, \theta)$ is the PDF of a random variable, y, conditional on the parameters θ. The formula for the joint density of n identical and separately disturbed observations is:

$$\prod_{i=1}^{N} f(y_i, \theta) = L(\theta|y) \tag{23}$$

$f(yi, \theta)$ is computed as below:

$$f(y_i, \theta) = f(y_i, \mu|\sigma^2 = 1) = \frac{e^{-\frac{1}{2}\left[\frac{y_i-\mu}{\sigma}\right]^2}}{\sigma\sqrt{2\pi}} \tag{24}$$

It is customary to work using the Log-Likelihood function:

$$L(\theta|y) = \sum_{i=1}^{N} In\left(\frac{e^{-\frac{1}{2}\left[\frac{y_i-\mu}{\sigma}\right]^2}}{\sigma\sqrt{2\pi}}\right) \tag{25}$$

For a random variable mean y, the maximum log-likelihood value can be computed by varying the value of μ as shown.

The Akaike Information Criteria (AIC) (Akaike, 1974: 19):

$$AIC = 2D + n.In - \left(\frac{\sum_{i=1}^{n}[C_{o,i}-C_{p,i}]^2}{n}\right) - 2CS. \tag{26}$$

The Bayesian Information Criteria (BIC) (Schwarz, 1978):

$$BIC = D.\ In + n.In - \left(\frac{\sum_{i=1}^{n}[C_{o,i}-C_{p,i}]^2}{n}\right) - 2CS. \tag{27}$$

RESULTS

To model the relationship between drought characteristics values, the researchers in this study utilized the MvCAT. MvCAT uses a Bayesian framework with a Gaussian probability

function to determine the copula parameters (Sadegh et al., 2017: 53). In Table 2, the relationship between drought characteristics was evaluated according to various correlation coefficient values. Accordingly, all correlation coefficients produce significant results around 95%. Moreover, the highest correlation coefficient was obtained with the Spearman rank correlation, while the lowest was obtained with the Kendall rank correlation.

Table 2: Evaluate dependence between the two input variables

Correlation type	Correlation Coefficient	p value	Significant at 5%
Kendall rank	0.778	0.000	Yes
Spearman's rank-order	0.9203	0.000	Yes
Pearson product-moment	0.9181	0.000	Yes

To determine the critical level or return period of an extreme event, distribution functions must be fitted as usual (Sadegh et al., 2018: 45). a range of continuous marginal distribution functions was used to determine which model best fits the drought characteristics. There are many distribution functions, including the Beta, Birnbaum-Saunders, Exponential, Extreme value, Gamma, Generalized extreme value, Generalized Pareto, Inverse Gaussian, Logistic, Log-logistic, Log-normal Nakagami, Normal, Rayleigh, Rician, t location-scale, and Weibull distributions. Figure 2 indicates the probability density function and QQ plot probability distributions that best fit the drought durations. The probability density function and QQ plot probability distributions that correspond most accurately with the severity of drought are illustrated in Figure 3.

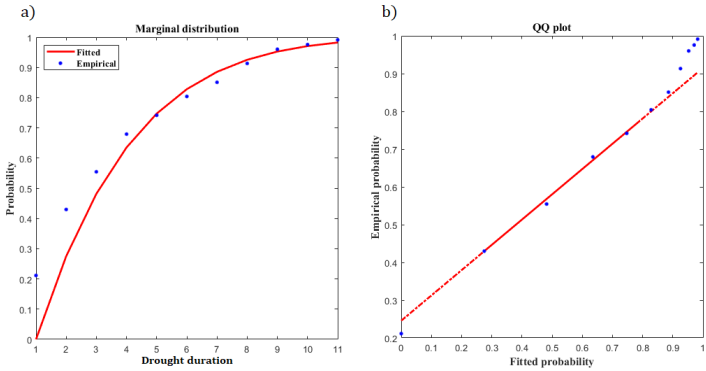


Figure 2: Distributions fitted for drought durations

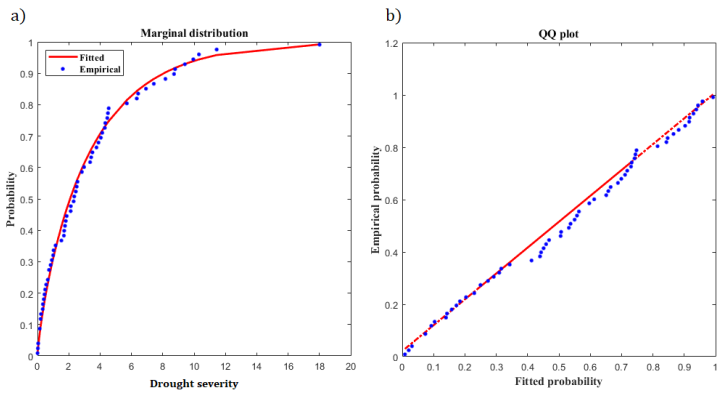


Figure 3: Distributions fitted for drought severities

Table 3 shows the drought characteristics, various distributions of the fitted distribution functions and the parameters of these distributions. Accordingly, while the Generalized Pareto distribution fits the drought periods the best, the Exponential distribution fits the drought intensities the best.

Table 3: Adapted PDF and parameters for drought characteristics

	Fitted distribution	Parameter 1	Parameter 2	Parameter 3
Drought duration	Generalized Pareto	k : -0.1294	sigma :3.174	Theta:1
Drought severity	Exponential	mu :3.3164		

The ranking of the various copula families used was evaluated in Table 4 according to various criteria such as likelihood, AIC, and BIC. these criteria generally showed the same result in determining the most suitable copula class. Accordingly, the best among 26 different copula classes were determined as Clayton, Frank, Gaussian, Gumbel, Joe, Plackett, AMH, respectively.

Table 4: Ranking of the 7 best fitted copula functions

Rank	Max-Likelihood	AIC	BIC
1	**Clayton**	**Clayton**	**Clayton**
2	Frank	Frank	Frank
3	Gaussian	Gaussian	Gaussian
4	Gumbel	Gumbel	Gumbel
5	Joe	Joe	Joe
6	Plackett	Plackett	Plackett
7	AMH	AMH	AMH

Note: Bold characters indicate optimum Copula function.

Table 5 shows the 95% confidence limits of Copula families. Accordingly, it is noteworthy that the parameters determined according to the MCMC technique are generally within the 95% Confidence limits. In accordance, for each selected copula family, NSE and RMSE values are used to present a 95% confidence range of parameter values through Local and MCMC techniques.

Table 5: The 95% confidence interval of the predicted parameters of the selected optimum copula classes

Estimated copula parameters					
Copula Name	**RMSE**	**NSE**	**Parameter 1 Local**	**Parameter 1 MCMC**	**Parameter 1 95% Confidence Range**
Gaussian	0.5912	0.9334	0.7532	0.9895	[0.9135,0.9981]
Clayton	0.5868	0.9344	0.0272	17.5451	[8.5366,34.2242]
Frank	0.5894	0.9338	10.7342	33.5019	[16.2296,34.7970]
Gumbel	0.5922	0.9332	1.5901	35	[6.3357,34.1822]
AMH	0.9349	0.8335	1	1	[0.8863,0.9993]
Joe	0.5927	0.9331	34.9992	35	[10.2345,34.7094]
Plackett	0.6467	0.9204	35	34.9991	[22.2370,34.8812]

The hydrological drought characteristics were modeled using the 26 bivariate copula functions in MvCAT. The following functions were used: In order: Gaussian, t, Clayton, Frank, Gumbel, Independence, Ali-Mikhail-Haq (AMH), Joe, Farlie-Gumbel-Morgenstern (FGM), Marshall-Olkin, Fischer-Hinzmann, Roch-Alegre, Fischer-Kock, BB1, BB5, Gumbel-Barnett, Plackett, Cuadras-Auge, Raftery, Shih-Louis, Linear-Spearman, Cubic, Burr, Nelsen, Galambos and Tawn copulas (Sadegh et al., 2017: 53).

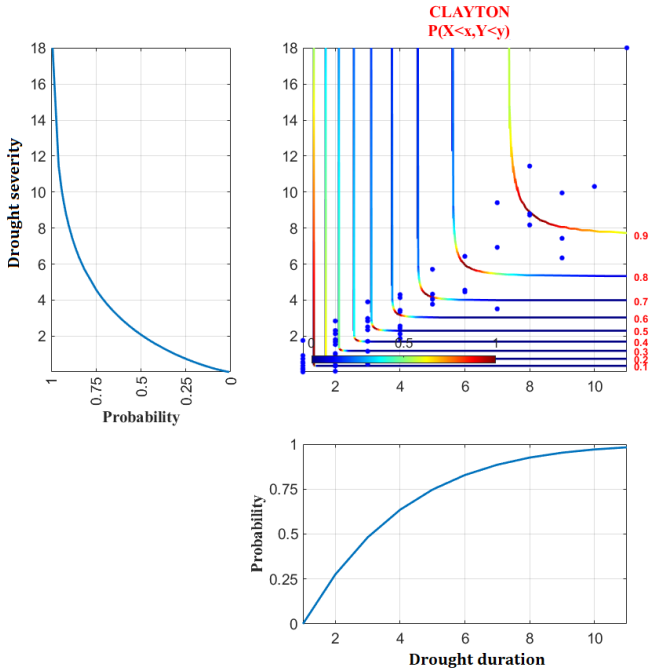


Figure 4: The Drought characteristic's marginal cumulative distributions and the joint probability

Figure 4 illustrates the marginal cumulative distributions of drought characteristics and the joint probability isolines that are derived from them. The changes in the probabilistic values of drought duration and severity variables, as well as the combined probabilities obtained from Clayton copula using these two variables, are also displayed.

The return periods of drought characteristics are examined in both univariate and multivariate contexts in Figure 5. The visualization of joint probability and multivariate return period isolines involves a color-coded scheme where low densities and high densities are used to represent the data. To facilitate the interpretation of the data, the joint probability density values are scaled to a range of 0 to 1, making it easier to visualize. In the representation, dots are used to indicate the observed

pairs of drought duration and associated drought severities. To put it another way, the blue dots on the map are employed to submit the frequency of drought events, and the isolines are colored to provide information about the likelihood of various combinations of drought duration and severity. According to this, when the return periods of its only variability are evaluated, it can be said that 10-year drought severities occur once in 25 years and 15-year drought severities occur once in a century. In addition, while droughts with a duration of 10 months are seen approximately once in 35 years, droughts occur for 50 years or more in the period above this. When the convergence probabilities are evaluated, it can be said that droughts with duration or severity of 2 and a half occur every 2 years, while droughts with duration or severity of 6 occur in 5 droughts with duration and severity of 7, and droughts of 10 severity or 10 duration occur approximately every 25 years.

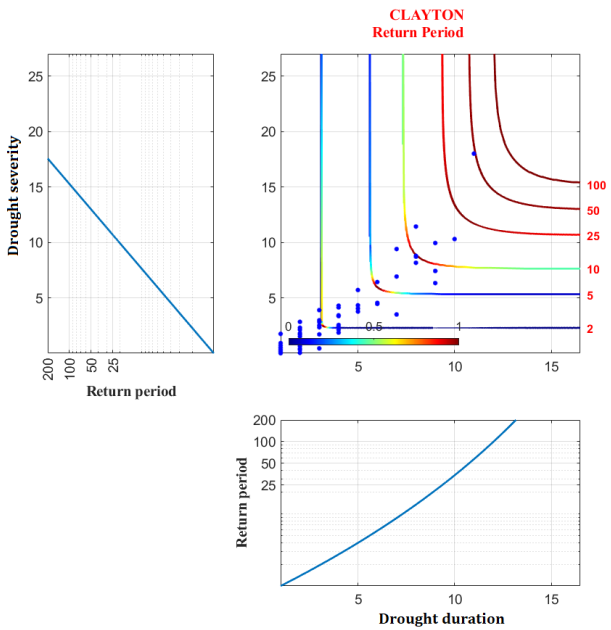


Figure 5: Univariate and multivariate return periods of drought characteristics

DISCUSSION

Katipoğlu and Can (2018: 25) applied the copula-based drought characteristic frequency analysis approach to determine the recurrence intervals. Among the possible copulas, the most suitable Gumbel copula was found for drought analysis. By comparing the Bayes Information Criteria and Akaike Information Criteria of the duration and severity of the drought, the best copula class was Gumbel for q=0.5 and q=0.1 cutoff levels, and the best copula family was Frank for q=0.5. The outputs of the Katipoğlu and Can (2018: 25) study are similar in terms of showing the Frank and Gumbel copula family, which are in the first 4 classes of copula functions that best fit the hydrological drought duration and severity in the Yeşilırmak River. However, due to the difference in the drought analysis technique used, Clayton copula, which is the best copula class, may cause weaker adaptation than Frank and Gumbel.

CONCLUSION

Within the scope of this study, MvCAT was applied to model the relationship between hydrological drought characteristics in stream gauging station 1401 located in the Yeşilırmak basin. The primary outputs of the study are listed as follows:

- Generalized Pareto and Exponential distributions were found to best match the duration and severity of hydrological droughts, respectively.
- The feasibility of the hybrid MCMC optimization technique for numerical estimation of copula parameters of hydrological drought characteristics is demonstrated.
- Various copula functions were compared in modeling hydrological drought characteristics and Clayton copula was found to be optimum in modeling hydrological drought characteristics.

For future studies, it is recommended to determine the meteorological and agricultural drought recurrence intervals and to make a drought risk calculation and to put into practice a basin-based drought management plan.

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REFERENCES

Ahmadi, F., Radmaneh, F., Sharifi, M.R., Mirabbasi, R. (2018). Bivariate Frequency Analysis of Low Flow Using Copula Functions (Case Study: Dez River Basin, Iran). *Environ Earth Sci* 77, 643.

Akaike, H. (1974). A New Look at Statistical Model Identification, IEEE T. *Autom. Control*, 19, 716–723.

Al-Faraj, F.A., Scholz, M., and Tigkas, D. (2014). Sensitivity of Surface Runoff to Drought and Climate Change: Application for Shared River Basins. *Water*, *6*(10), 3033-3048.

Bárdossy, A. (2006). Copula-Based Geostatistical Models for Groundwater Quality Parameters. *Water Resources Research* 42 (11).

Bonaccorso, B., Cancelliere, A., Rossi, G. (2003). An Analytical Formulation of Return Period of Drought Severity. *Stochastic Environmental Research and Risk Assessment* 17, 157–174.

Chacon-Hurtado, J.C., Alfonso, L., Solomatine, D.P. (2017). Rainfall and Streamflow Sensor Network Design: A Review of Applications, Classification, and A Proposed Framework. *Hydrology and Earth System Sciences* 21, 3071–3091.

Chen, L., Singh, V.P., Guo, S.L., Mishra, A.K, Guo, J. (2013) Drought Analysis Using Copulas. *J Hydrol Eng.* 18, 797-808.

Chen, L., Singh, V.P., Shenglian, G., Hao, Z., Li, T. (2012). Flood Coincidence Risk Analysis Using Multivariate Copula Functions. *Journal of Hydrologic Engineering* 17, 742–755.

Clayton, D.G. (1978). A Model for Association in Bivariate Life Tables and Its Application in Epidemiological Studies of Familial Tendency in Chronic Disease Incidence. *Biometrika*, *65*(1), 141-151.

Esit, M. and Yuce, M.I. (2023). Copula-Based Bivariate Drought Severity and Duration Frequency Analysis Considering Spatial–Temporal Variability in The Ceyhan Basin, Turkey. *Theoretical and Applied Climatology*, *151*(3-4), 1113-1131.

Hesami Afshar, M., Sorman, A.U., Yilmaz, M.T. (2016). Conditional Copula-Based Spatial–Temporal Drought Characteristics Analysis—A Case Study over Turkey. Water 8, 426.

Hezarani, A.B., and Keskin, A.U. (2021). Comparison of Four Precipitation Based Meteorological Drought Indices in Yesilirmak Basin, Turkey.

Ismail, T., Alamgir, M.M., Noor, M. (2018). Bivariate Frequency Analysis of Flood Variables Using Copula in Kelantan River Basin. *Malaysian Journal of Civil Engineering* 30, 395–404.

Kao, S.C., Govindaraju, R.S. (2010). A Copula-Based Joint Deficit Index for Droughts. *Journal of Hydrology* 380, 121–134.

Katipoglu, O.M., and Can, I. (2018). Estimation of Drought Risks Using Archimedean Copulas In The Karasu River, Turkey. *Feb Fresenius Environ Bull*, 25, 9799.

Katipoğlu, O.M. (2023). Prediction of Streamflow Drought Index for Short-Term Hydrological Drought in The Semi-Arid Yesilirmak Basin Using Wavelet Transform and Artificial İntelligence Techniques. *Sustainability*, *15*(2), 1109.

Khan, M. A., Faisal, M., Hashmi, M. Z., Nazeer, A., Ali, Z., and Hussain, I. (2021). Modeling Drought Duration and Severity Using Two-Dimensional Copula. *Journal of Atmospheric and Solar-Terrestrial Physics*, 214, 105530.

Kim, T. W., Valdes, J. B., and Aparicio, J. (2006). Spatial Characterization of Droughts in the Conchos River Basin Based on Bivariate Frequency Analysis. *Water international*, 31(1), 50-58.

Kim, T.W., Valdés, J.B., Yoo, C. (2003). Nonparametric Approach for Estimating Return Periods of Droughts in Arid Regions. *Journal of Hydrologic Engineering* 8, 237–246.

Kumar, P. (2010). Probability Distributions and Estimation of Ali-Mikhail-Haq Copula. *Applied Mathematical Sciences*, *4*(14), 657-666.

Li, C., Singh, V.P., and Mishra, A.K. (2013). A Bivariate Mixed Distribution with a Heavy-Tailed Component and Its Application to Single-Site Daily Rainfall Simulation. *Water Resources Research*, *49*(2), 767-789.

Li, Z., Shao, Q., Tian, Q., and Zhang, L. (2020). Copula-Based Drought Severity-Area-Frequency Curve and Its Uncertainty, A Case Study of Heihe River Basin, *China. Hydrology Research*, 51(5), 867-881.

Liu, X.F., Wang, S.-X., Zhou, Y., Wang, F.-T., Yang, G., Liu, W.L. (2016). Spatial Analysis of Meteorological Drought Return Periods in China Using Copulas. *Nat Hazards* 80, 367–388

Mishra, A.K., Singh, V.P. (2010). A Review of Drought Concepts. Journal of Hydrology 391, 202–216.

Nalbantis, I., and Tsakiris, G. (2009). Assessment of Hydrological Drought Revisited. *Water resources management*, *23*, 881-897.

Reddy, M.J., Singh, V.P. (2014). Multivariate Modeling of Droughts Using Copulas and Meta-Heuristic Methods. *Stochastic environmental research and risk assessment.* 28(3), 475-489

Sadegh, M., Moftakhari, H., Gupta, H.V., Ragno, E., Mazdiyasni, O., Sanders, B., . . . AghaKouchak, A. (2018). Multihazard Scenarios For Analysis of Compound Extreme Events. Geophysical Research Letters, 45(11), 5470-5480.

Sadegh, M., Ragno, E., and AghaKouchak, A. (2017). Multivariate Copula A nalysis Toolbox (MvCAT): Describing Dependence and Underlying Uncertainty Using a Bayesian Framework. *Water Resources Research,* 53(6), 5166-5183.

Salas, J.D., Fu, C., Cancelliere, A., Dustin, D., Bode, D., Pineda, A., Vincent, E., (2005). Characterizing the Severity and Risk of Drought in the Poudre River, Colorado. *Journal of Water Resources Planning and Management* 131, 383–393.

Salvadori, G., Michele, C.D., Kottegoda, N.T., Rosso, R. (2007). Extremes in Nature: An Approach Using Copulas, *Water Science and Technology Library*. Springer Netherlands (Vol. 56).

Schwarz, C. J. (2011). Sampling, Regression, Experimental Design and Analysis for Environmental Scientists, Biologists, and Resource Managers. *Department of Statistics and Actuarial Science*, Simon Fraser University, 57.

Schwarz, G. (1978). Estimating The Dimension of a Model. *The annals of statistics*, 461-464.

Serinaldi, F., Bonaccorso, B., Cancelliere, A., Grimaldi, S. (2009). Probabilistic Characterization of Drought Properties Through Copulas. *Physics and Chemistry of the Earth, Parts A/B/C, Recent developments of statistical tools for hydrological application* 34, 596–605.

She, D., Xia, J. (2018). Copulas-Based Drought Characteristics Analysis and Risk Assessment Across the Loess Plateau of China. *Water Resour Manage* 32, 547–564.

Shiau, J.T. (2006). Fitting Drought Duration and Severity with Two-Dimensional Copulas. Water Resour Manag. 20, 795-815.

Shiau, J.T., Feng, S., Nadarajah, S. (2007). Assessment of Hydrological Droughts for The Yellow River, China, Using Copulas. *Hydrological Processes* 21, 2157–2163.

Shiau, J.T., Modarres, R. (2009). Copula-Based Drought Severity-Duration-Frequency Analysis in Iran. *Meteorological Applications* 16, 481–489.

Shiau, J.T., Modarres, R., Nadarajah, S. (2012). Assessing Multi-site Drought Connections in Iran Using Empirical Copula. *Environmental modeling and assessment.* 17(5), 469-482

Shiau, J.T., Shen, H.W., (2001). Recurrence Analysis of Hydrologic Droughts of Differing Severity. *Journal of Water Resources Planning and Management* 127, 30– 40.

Sklar, M., (1959). Fonctions de Répartition À N Dimensions Et Leurs Marges. Université Paris 8.

Song, S., Singh, V.P. (2010). Frequency Analysis of Droughts Using the Plackett Copula and Parameter Estimation by Genetic Algorithm. *Stoch Environ Res Risk Assess* 24, 783–805.

Thyer, M., Renard, B., Kavetski, D., Kuczera, G., Franks, S. W., and Srikanthan, S. (2009). Critical Evaluation of Parameter Consistency and Predictive Uncertainty in Hydrological Modeling: A Case Study Using Bayesian Total Error Analysis. *Water Resources Research*, *45*(12).

Tosunoglu, F., and Can, I. (2016). Application of Copulas for Regional Bivariate Frequency Analysis of Meteorological Droughts in Turkey. Natural Hazards, 82, 1457-1477.

Tosunoglu, F., Kisi, O. (2016). Joint modelling of annual maximum drought severity and corresponding duration. *Journal of Hydrology* 543, 406–422.

Vazifehkhah, S., Tosunoglu, F., Kahya, E. (2019). Bivariate Risk Analysis of Droughts Using a Nonparametric Multivariate Standardized Drought Index and Copulas. *Journal of Hydrologic Engineering* 24, 05019006.

Wang, C. (2016). A Joint Probability Approach for Coincidental Flood Frequency Analysis at Ungauged Basin Confluences. *Natural Hazards* 82(3), 1727-1741.

Wang, F., Wang, Z., Yang, H., Zhao, Y., Zhang, Z., Li, Z., Hussain, Z. (2019). Copula-Based Drought Analysis Using Standardized Precipitation Evapotranspiration Index: A Case Study in the Yellow River Basin, China. *Water* 11(6), 1298

Wong, G., Lambert, M.F., Metcalfe, A.V. (2008). Trivariate Copulas for Characterisation of Droughts. *Anziam Journal.* 49, 306-23.

Yee, K. C., Suhaila, J., Yusof, F., and Mean, F. H. (2014, July). Bivariate Copula in Fitting Rainfall Data. In *AIP conference proceedings* (Vol. 1605, No. 1, pp. 986-990). *American Institute of Physics.*

Yoo, J., Kim, D., Kim, H., Kim, T.-W., (2016). Application of Copula Functions to Construct Confidence Intervals of Bivariate Drought Frequency Curve. Journal of Hydroenvironment Research 11, 113–122.

Zeybekoglu, U., Hezarani, A.B., and Keskin, A.U. (2023). Comparison of Four Precipitation Based Meteorological Drought Indices in The Yesilirmak Basin, Turkey. *Quarterly Journal of the Hungarian Meteorological Service*, 127(1), 123-142.