Memristor Based PD Controller Design and Application on a Ball and Beam Control System

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Abstract **- In this study, a memristor based PD (Mem-PD) controller is designed and the performance of the controller is tested in the simulation model of the beam and ball (Ball and Beam) control system. In the proposed control structure, the position of the ball in the primary part called the outer loop is controlled by changing the angle of the beam. In the secondary part, called the inner loop, voltage is generated that will determine the proper position angle of the servomotor to adjust the position of the beam. In addition, the use of Memristor, which has attracted increasing attention in recent years, in the field of control systems has been tested. The simulation results show that the designed controller successfully performed reference tracking.**

Keywords **– PD Control, Memristor, Ball and Beam, MemPD.**

I. INTRODUCTION

MEMRISTOR, first proposed by Leon Chua in 1971 as a lost circuit element, was defined as a memory resistor [1]. After the memristor was produced in HP laboratories in 2008, it became more popular and its popularity increased [2]. In order to create this circuit element, first, silicon dioxide or titanium dioxide is obtained by using titanium and silicon semiconducts and oxygen atoms, and by using any of these compounds, the internal structure of the memristor is formed. As the current flows through the memristor, the oxygen spaces in the doped titanium dioxide layer slide towards the direction of the current, that is, pure titanium dioxide, as a result, the pure titanium dioxide range narrows. The spacing length of doped titanium dioxides increases and the boundary between TiO_2 film layers changes, causing the resistance to change. In the absence of current, the doped and pure TiO_2 layers maintain their gap levels. In the opposite case, that is, with the negative flow of current, the oxygen spaces move towards the doped layer. Then the pure TiO_2 layer increases its width, causing an increase in resistance. Until today, many studies have been carried out in different fields by using memristor models and emulator circuits [3-9]. In addition, studies were using memristor in the field of control systems have become widespread in recent years. Delgado stated in his study that the Memristor can be used as an adjustable gain for closed loop systems. Also it has been stated that the memristor acts as a linear resistor at high frequencies [10]. Wnag et al. [11] presented a mathematical model of an M-PID controller by replacing a resistance of a conventional PID control circuit with a memistor. They stated that the M-PID controller behaves faster and stronger in terms of dynamic performance compared to the traditional PID device. Wang et al. [12] addressed the problem of finite-time stabilization of memistor-based delayed neural networks (MDNNs) through two control approaches in their study. Lu et al. [13] presented self-adjusting PID circuits with voltage and current controlled memristor in their work. In addition, simulation and experimental results are presented to show that the structure with memristor has a more suitable robustness than traditional PID controller. In a study in which parameter update rules of voltage and current controlled memristors were discussed [14]; The performance of the closed loop system (DC / DC power convertor) was tested under the control of the memristor based adaptive PI controller. With simulations and experimental results, it is stated that the controller parameters can vary with the operating point of the controlled system compared to the traditional PI controller, so the proposed controller is more robust and is suitable for scenarios with large power supplies and load disturbances. In another controller design application made with memristor [15]; Simulation study of temperature profile tracking control in heat flow system using memristor based PI controller is presented. The success of the Mem-PI controller, which is obtained by using memristor instead of the gain resistor in traditional PI controller, is presented with simulation results.

Ball and Beam experimental setup is a system widely used in many university laboratories to show students the applications of control algorithms [16-19]. The purpose of this system is to control the position of the ball on the beam.

In this study, in order to test the usability of the Memristor in the field of control systems, the traditional PD controller was redesigned using a memristor and the Mem-PD controller obtained was applied to the Ball and Beam system. Simulation results are presented to demonstrate the recommended controller success.

II. MODELLING OF BALL AND BEAM SYSTEM

The ball and beam experimental setup system was designed by Quanser, and information about the system is clearly described in their documentation [20]. The schematic diagram of the ball and beam system is shown in Fig. 1. Here m_b is the mass of the ball, r_b is the radius of the ball, γ_b is the angle of the ball, x is the position of the ball, α is the angle with the beam axis, g_b is gravitational acceleration and J_b is momentum.

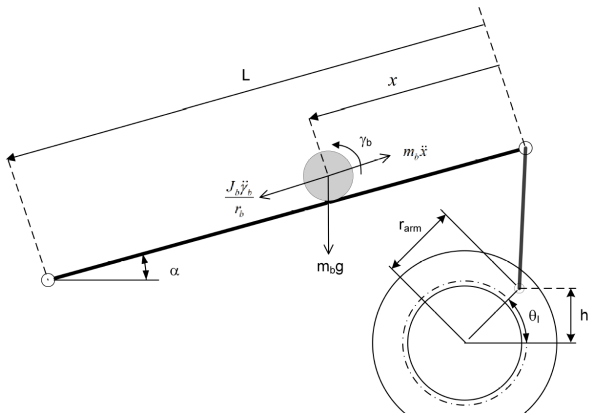


Figure 1: Free body diagram of the Ball and Beam system.

In the light of Ref. [20] the linearized acceleration equation of the ball can be obtained as follows.

$$\ddot{x}(t) = \frac{m_b g \sin\alpha(t) r_b^2}{m_b r_b^2 + J_b}. \quad (1)$$

The relationship between the angle of the servo motor $\theta_l(t)$ and the length of the beam L_{beam} can be expressed as follows, considering the servo dynamics.

$$\sin\alpha(t) = \frac{\sin\theta_l(t) r_{arm}}{L_{beam}}. \quad (2)$$

Here, the r_{arm} is the distance between the plated joint and the output gear shaft. The relation mentioned in (2) can be added to the nonlinear motion equation found in [20]. The final form of the dynamic equation of the ball beam system after mathematical manipulation is as follows.

$$\ddot{x}(t) = \frac{m_b g \theta_l(t) r_{arm} r_b^2}{L_{beam}(m_b r_b^2 + J_b)}. \quad (3)$$

III. Controller Design

If x_d is the desired position and x is the current position, position tracking error of the Ball and Beam system for the controller design can be defined as follows.

$$e(t) = x(t) - x_d(t). \quad (4)$$

The PD controller designed using op-amp is shown in Fig. 3. The capacitor in the circuit is used to store charge and represents the differential equation of the input. In the PD controller circuit, the output $u(t)$ can be written as below;

$$u(t) = -\frac{R_2}{R_1} e(t) - R_2 C \frac{d}{dt} e(t). \quad (5)$$

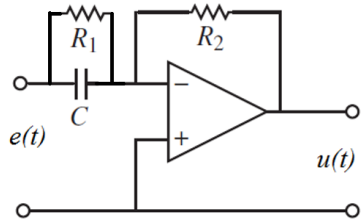


Figure 2: Schematic diagram for PD controller using operational amplifier.

The following statement can express the relationship between the current and voltage of the Memistor.

$$V(t) = R(t) i(t) = \frac{d\varphi}{dQ} i(t). \quad (6)$$

Here, Q and φ denote charge and flux, respectively. The ratio of flux to charge is called memristance and refers to the resistance of the memristor. The relationship between current and voltage in TiO_2 memistor has been proposed by the HP research group [7] as follows.

$$V(t) = \left[R_{ON} \frac{X(t)}{D} + R_{OFF}\left(1 - \frac{X(t)}{D}\right) \right] i(t). \quad (7)$$

In the above equation, R_{ON} and R_{OFF} refer to high and low resistance values, respectively. X and D indicate and doped area and the thickness of the TiO_2 memristor, respectively. Resistance value, which depends on the direct change of the X value, is defined as:

$$\frac{dX(t)}{dt} = \mu_v \frac{R_{ON}}{D} i(t). \quad (8)$$

Here μ_v refers to the mobility of the memistor. Memistors show a stuck hysteresis voltage-current relationship when a sinusoidal signal is applied, and there is an inverse ratio between frequency and nonlinearity as shown in Fig. 3.

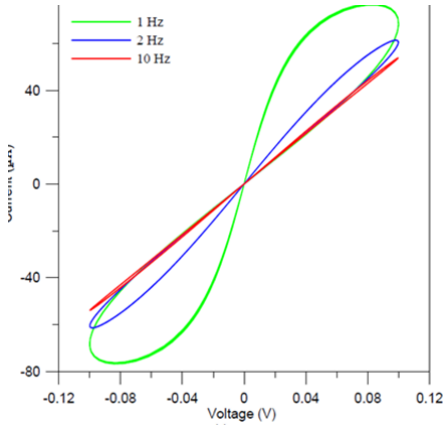


Figure 3: Memistor current-voltage relationship for various frequencies [7].

To achieve the Mem-PD controller, we can replace R_2 with

a memistor in the circuit given in Fig. 2. In this way, the circuit belonging to the Mem-PD controller will be as in the Fig. 4 below.

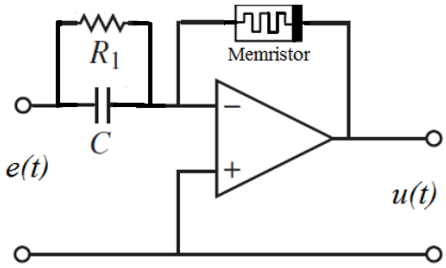


Figure 4: The Mem-PD controller circuit

Considering (5), (6) and (7), if the control signal *u(t)* in the circuit given in Fig. 4 is rewritten;

$$u(t) = -\frac{\left[R_{ON}\frac{X(t)}{D} + R_{OFF}(1-\frac{X(t)}{D})\right]}{R_1}e(t) - \left[R_{ON}\frac{X(t)}{D} + R_{OFF}(1-\frac{X(t)}{D})\right]C\frac{d}{dt}e(t). \quad (9)$$

Considering Ref. [20], the block diagram of the position control of the Ball and Beam system with Mem-PD control approach is given in Figure 5.

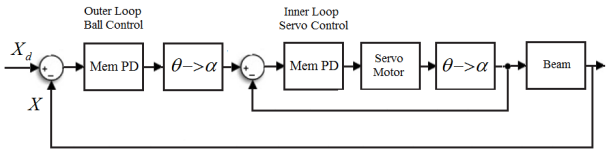


Figure 5: Position control of the Ball and Beam system using Mem-PD controller

IV. Simulation

In this section, to test the performance of both the conventional PD controller and the designed Mem-PD controller, a square wave position reference signal with sudden changes was applied to the ball and beam system and the simulation results were obtained and given in Fig. 6-8.

When the Fig. 6 showing the position of the ball is examined, it is observed that both controllers show approximately the same performance. However, according to the zoomed image, it is seen that the Mem-PD controller reached the reference 0.4 seconds earlier than the PD controller.

In the graph showing the servo angles and given in Fig. 7, especially during the sudden changes in the 5th and 10th seconds, the PD controller has 3% more overshot than the Mem-PD controller.

Fig. 8 shows the control signals of both control devices. Although similar signals are generated, the amplitude of the control signal of the PD controller is 2% more than the control signal amplitude produced by the Mem-PD at the sudden change moments.

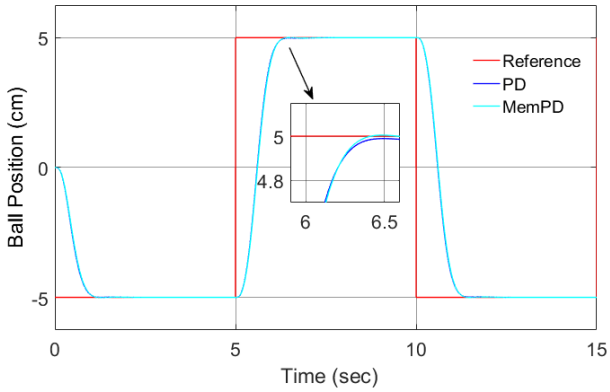


Figure 6: Simulation result of Ball positon of the Ball and Beam system using PD and Mem-PD controllers

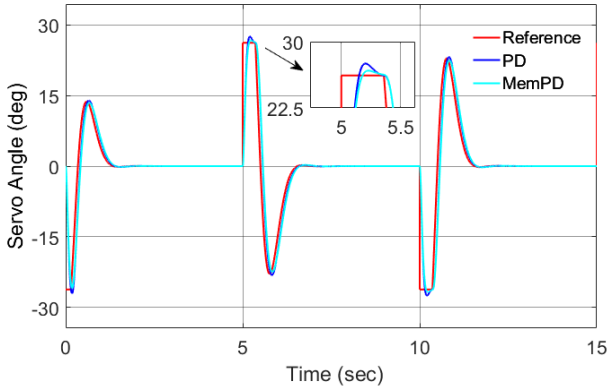


Figure 7: Servo angles for PD and Mem-PD controller

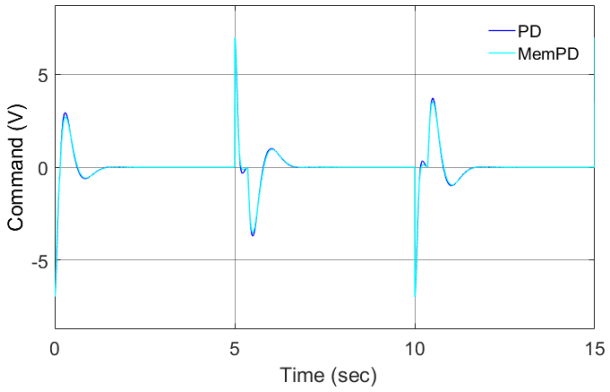


Figure 8: Command signals for PD and Mem-PD controllers

V. Conclusion

In this study, a memristor based PD controller is proposed. The control structure was achieved by using a memistor instead of the gain resistor in the traditional PD controller circuit. The performance of the designed Mem-PD controller was compared with the traditional PD controller, and the simulation results showed that the proposed controller was successful in both reference tracking and error recovery. In

future work, it is planned to test this controller in a real time experimental setup.

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